The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Final Exam

Date: May 14, 2009 Course: EE 313 Evans

Name: ____
Last, First

- The exam is scheduled to last 3 hours.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- **Power off all cell phones and pagers**
- You may use any standalone computer system, i.e. one that is not connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise. If you cite a reference, then please provide a page number and the quote you are using.**

Problem	Point Value	Your score	Topic
1	10		Differential Equation Rhythm
2	10		Differential Equation Blues
3	10		Stability
4	10		Convolution in Two Domains
5	10		Sampling in Continuous Time
6	15		Discrete-Time Filter Analysis
7	15		Discrete-Time Filter Design
8	10		Sinusoidal Amplitude Modulation
9	10		Sinusoidal Amplitude Demodulation
Total	**100**		

Final Exam Problem 1. Differential Equation Rhythm. 10 points.

Consider a continuous-time system with input $x(t)$ and output $y(t)$ governed by the differential equation

$$\frac{d^2}{dt^2} y(t) + 7\frac{d}{dt} y(t) + 10 y(t) = x(t)$$

for $t \geq 0^+$.

(a) What are the characteristic roots of the differential equation? 2 points.

(b) Find the zero-input response assuming non-zero initial conditions. Please leave your answer in terms of C_1 and C_2. 4 points.

(c) Find the zero-input response for the initial conditions $y(0^+) = 0$ and $y'(0^+) = 1$. 4 points.

Final Exam Problem 2. Differential Equation Blues. 10 points.

Consider a continuous-time linear time-invariant (LTI) system with input $x(t)$ and output $y(t)$ governed by the differential equation

$$\frac{d^2}{dt^2} y(t) + 7 \frac{d}{dt} y(t) + 10 y(t) = x(t)$$

for $t \geq 0^-$.

(a) What is the transfer function in the Laplace domain? 2 points.

(b) What are the values of the poles and zeroes of the transfer function? 2 points.

(c) What is the region of convergence for the transfer function? 2 points.

(d) What is the step response of the system in the time domain? 4 points.

Final Exam Problem 3. Stability. 10 points.

In this problem, the input signal is denoted by $x(t)$ and the output signal is denoted by $y(t)$. The input-output relationship of a system is defined as

$$\frac{d^2}{dt^2} y(t) + 4\frac{d}{dt} y(t) + K y(t) = x(t)$$

where K is an adjustable gain that can take any real value. By adjusting K, one can change the time response and frequency response of the system. Assume all initial conditions to be zero.

Assume that K is a constant (but of unknown value) and the system is linear and time-invariant.

(a) What are the pole locations? Express your answer in terms of K. 3 points.

(b) For what values of K is the system bounded-input bounded-output stable? 2 points.

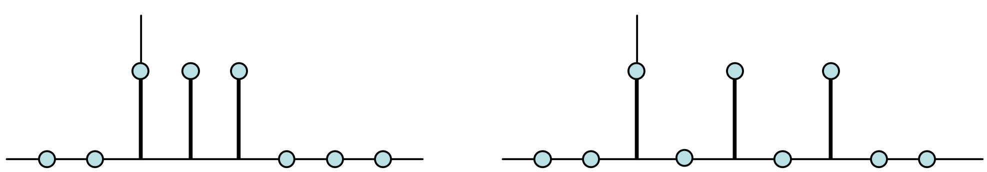
(c) Plot the pole locations in the Laplace domain as K varies. 2 points.

(d) Describe the frequency selectivity of the system (lowpass, highpass, bandpass, bandstop, notch, or allpass) for all possible values of K for which the system is bounded-input bounded-output stable. 3 points.

Final Exam Problem 4. Convolution in Two Domains. 10 points.

(a) In continuous time, convolve the unit step function $u(t)$ and the signal $\delta(t) - \delta(t-T)$, where $\delta(t)$ is the Dirac delta functional and T is a positive real number. 5 points.

(b) Consider a causal discrete-time linear time-invariant system. For input $x[n]$ given below, the system gives output $y[n]$ below. What is the impulse response $h[n]$ of the system? Both $x[n]$ and $y[n]$ are of finite extent. 5 points.



Final Exam Problem 5. Sampling in Continuous Time. 10 points.

Sampling of an analog continuous-time signal $f(t)$ can be modeled in continuous-time as

$$y(t) = f(t)\, p(t)$$

where $p(t)$ is the impulse train defined by

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

such that T_s is the sampling duration. The Fourier series expansion of the impulse train is

$$p(t) = \frac{1}{T_s}\left(1 + 2\cos(\omega_s\, t) + 2\cos(2\,\omega_s\, t) + \ldots\right)$$

where $\omega_s = 2\,\pi\, /\, T_s$.

(a) Plot the impulse train $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$. 2 points.

(b) Find $P(\omega)$, the Fourier transform of $p(t)$. 2 points.

(c) Express your answer for $P(\omega)$ in part (b) as an impulse train in the Fourier domain. 3 points.

(d) What is the spacing of the impulse train $P(\omega)$ with respect to ω? 3 points.

Final Exam Problem 6. *Discrete-Time Filter Analysis*. 15 points.

A causal discrete-time linear time-invariant filter with input $x[n]$ and output $y[n]$ is governed by the following difference equation:

$$y[n] = -0.8\, y[n-1] + x[n] + 1.25\, x[n-1]$$

(a) Draw the block diagram for this filter. 3 points.

(b) What are the initial conditions? What values should they be assigned? 3 points.

(c) Find the equation for the transfer function in the z-domain including the region of convergence. 3 points.

(d) Find the equation for the frequency response of the filter. 3 points.

(e) Describe the frequency selectivity of this filter as lowpass, bandpass, bandstop, highpass, notch, or allpass. Why? 3 points.

Final Exam Problem 7. *Discrete-Time Filter Design*. 15 points.

Digital Subscriber Line (DSL) systems transmit voice and data over a telephone line using frequencies from 0 Hz to 1.1 MHz. DSL systems use a sampling rate of 2.2 MHz.

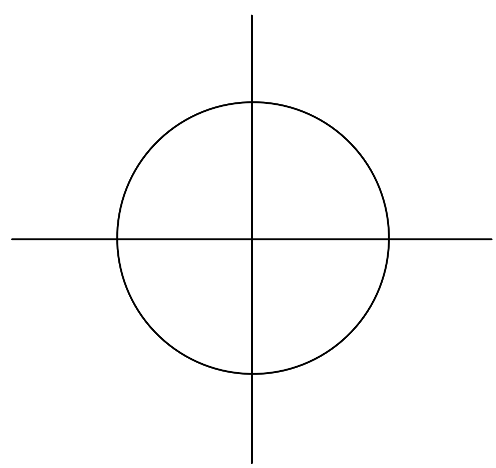
Consider an AM radio station that has a carrier frequency of 550 kHz, has a transmission bandwidth of 10 kHz, and is interfering with DSL transmission.

Design a discrete-time filter *biquad* for the DSL receiver to reject the AM radio station but pass as much of the DSL transmission band as possible. A biquad has 2 poles and 0, 1, or 2 zeros.

(a) Is the frequency selectivity of the discrete-time IIR filter biquad lowpass, bandpass, bandstop, highpass, notch, or allpass? Why? 3 points.

(b) Give formulas for the locations of the poles and zeros of the biquad. 5 points.

(c) Draw the poles and zeros on the pole-zero diagram on the right. The circle has a radius of one. 4 points.



(d) Compute the scaling constant (gain) for the filter's transfer function. 3 points.

Final Exam Problem 8. Sinusoidal Amplitude Modulation. 10 points.

In practice, we cannot generate a two-sided sinusoid $\cos(2 \pi f_c t)$, but we can generate a one-sided sinusoid $\cos(2 \pi f_c t)\, u(t)$.

Consider a one-sided cosine $c(t) = \cos(2 \pi f_c t)\, u(t)$ where f_c is the carrier frequency (in Hz).

(a) By using the Fourier transforms of $\cos(2 \pi f_c t)$ and $u(t)$ from a lookup table, compute the Fourier transform of $c(t) = \cos(2 \pi f_c t)\, u(t)$ using Fourier transform properties. 3 points.

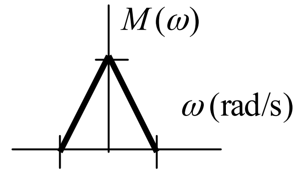
(b) Draw $|C(\omega)|$, the magnitude of the Fourier transform of $c(t)$. 3 points.

(c) Describe the differences between the magnitude of the Fourier transforms of a one-sided cosine and a two-sided cosine. What is the bandwidth of each signal? 4 points.

Lowpass
Filter

$-\omega_m$ ω_m

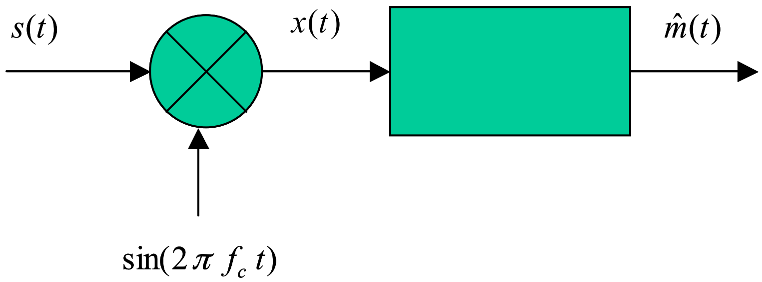
Final Exam Problem 9. Sinusoidal Amplitude Demodulation. 10 points.



A lowpass, real-valued message signal $m(t)$ with bandwidth f_m (in Hz) is to be transmitted using sinusoidal amplitude modulation

$$s(t) = m(t) \sin(2 \pi f_c t)$$

where f_c is the carrier frequency (in Hz) and $f_c >> f_m$. The receiver processes the transmitted signal $s(t)$ to obtain an estimate of the message signal, $\hat{m}(t)$, as follows:



Hence, $x(t) = s(t) \sin(2 \pi f_c t)$. $M(\omega)$ is plotted above to the upper right.

(a) Plot the Fourier transform of $s(t)$, i.e. $S(\omega)$. 4 points.

(b) Plot the Fourier transform of $x(t)$, i.e. $X(\omega)$. 4 points.

(c) Give the maximum passband frequency and the minimum stopband frequency for the lowpass filter to recover $m(t)$. 2 points.